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Anomalous Fluctuations in Random Walk Dynamics

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The anomalous dispersion of noninteracting particles randomly walking in a network is considered. It is shown that the existence of large dangling branches attached to a backbone induces a "$1/f$"-like behavior in the current autocorrelation function at low frequencies. The waiting times associated with dangling loops scale like $t^{-3/2}$. The size of the dangling branches provides a lower cutoff to the power law behavior. When the side branches are infinite, self-similar structures, the power law behavior persists up to a zero frequency. The currents we consider are created either by a bias on the random walk or by a current source. We consider both the total current, which is often referred to in the literature, and the current measured at endpoints of a specimen attached to a (model) battery. The differences and similarities between the two corresponding correlations are analyzed. In particular, we find that in the second case "$1/f$" noise exists only for large bias. When a statistical distribution of dangling branches is considered, we find that the largest power of frequency in the spectrum is 1.13. Much of our results are true when the dangling branches are replaced by "traps" having waiting time distributions that equal those of the branches. The waiting time associated with a power law distribution of dangling loops (m^{-x}: m is the length of the loop) scales like $t^{-1-(x/2)}$. However, it is shown that geometry alone can be responsible for the appearance of power laws in the spectra. Random geometry can be regarded as a model (or source) of random hopping times.

KEY WORDS: Random walk; anomalous diffusion; $1/f$ noise; current spectra; percolation.

1. INTRODUCTION

It is by now an established fact that many transport processes can be well described by random walk models.[1-7] These include such diverse

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phenomena as conduction in amorphous semiconductors[2,3,8,9] and diffusion in porous media.[10–14]

When conduction by electrically charged particles is considered, one should always worry about the interactions among the particles. However, in many cases these interactions can be safely neglected in constructing a transport model. A known case is that of hopping conduction in semiconductors, mentioned above. In general, when strong screening effects are present and the gas of electrons (or holes or ions, as the case may be) is dilute enough, an independent-particle model is expected to be valid, at least approximately. Even in metals, when one deals with phenomena on time scales that are much shorter than the inverse plasma frequency (and these are really very short times), the electrons can be supposed to be noninteracting.

In the present work, we analyze the transport properties on a network on which there are noninteracting random walkers. We investigate the role of bias as well as the effects of irregular geometry. For sake of definiteness, we employ an "electrical" language: we deal with "charges," "currents," and even "batteries." These notions should not be taken literally. Our work applies to neutral particles as well. In this case, the "battery" is merely a source of bias (and feedback) for the random walker, as is the mean velocity of the fluid in the process of diffusion in porous media.

One of the most intriguing phenomena observed in many physical systems is known as "$1/f$" noise.[15] Many theories have been proposed as explanations of this effect. It seems quite unclear, at present, whether "$1/f$" noise is a universal phenomenon, having a common (mathematical) formulation, or that each observed "$1/f$" behavior deserves a separate explanation. The work in this field ranges from that based on the theory of dynamical systems[16,17] to models of random hopping in systems with traps.[2,3,18,19] Since the present work deals with random walk on networks,[29,21] we are obviously close to the latter theories. The difference between the theory proposed here and the models that include traps is that we consider the role of geometry as a source of anomalous transport fluctuations.[22] We show below that a dangling branch in a network that can delay a random walker (and such objects are easy to construct) can mimic the effect of a trap. Conversely, given such a branch, one can replace it by a trap in a way that leaves the transport properties unchanged. In many systems, however, such geometric traps are the reality: percolation clusters[23] and porous media are typical examples. One can, of course, have both traps and dangling branches.

The main model to be analyzed below is that of a one-dimensional segment composed of discrete points each of which is atached to a dangling or side branch. We choose, for simplicity, a discrete space and time nearest neighbor hopping dynamics. The hopping may either be biased or unbiased. When unbiased hopping is discussed, it is assumed, for simplicity, that the hopping probability w is 1/4 (see also Ref. 22).

We consider two kinds of mechanisms that are responsible for creating a current through the network: either an external bias or a current source, injecting particles at a constant rate at one end of the system. One of the problems discussed below is the nature of the current measured by an external device.[24] We deal separately with the total current in the network, which is the quantity of interest in many investigations[2,3,6] and with the current flowing through an external device connected to the system.

The method of analysis of the models below is based on the approach developed in Refs. 20 and 21 and also employs results from a previous paper by us,[22] which we refer to as I. Our approach is fully analytic.

The structure of the paper is a follows. In Section 2 we compute the properties of current fluctuations and their corresponding spectra for the model described above, using different boundary conditions and bias values. In the section we assume that all dangling branches are alike. This assumption is lifted in Section 3, where we consider a statistical ensemble of dangling branches. This assumption is obviously more realistic than the one made in Section 2. Indeed, as we shall see below, it leads to a modification of the transport properties. Section 4 offers a brief summary and a discussion of the results.

2. CURRENTS, CORRELATIONS, AND SPECTRA

2.1. Total Current Correlations

The quantity of interest in this subsection is the *total* current in a sample.[2,3,8] This is a quantity commonly referred to in investigations of conduction processes; it has been measured, for example, in amorphous semiconductors.[3,8] We consider a simple system consisting of a straight segment with dangling loops (see Fig. 1). The dynamics of the particles on

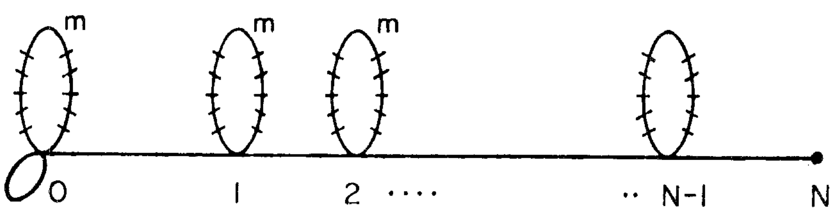

Fig. 1. The segment with loops (see text).

this network is that of hopping to nearest neighbors. The randomly walking particles are assumed to be noninteracting. This model has been described in I. As before, the process is assumed to be discrete in time (time is an integer describing the "number of steps"). Define N_c to be the number of carriers in the sample, e their effective charge, $v_i(t)$ the velocity of carrier i, and L the length of the sample. Then the instantaneous total current $I(t)$ per unit length is

$$I(t) = \frac{e}{L} \sum_{i=1}^{N_c} v_i(t) \tag{2.1}$$

The correlation function of the current fluctuations $C(t)$ is

$$C(t) = \overline{\delta I(0)\, \delta I(t)} \tag{2.2}$$

where $\delta I(t) = I(t) - \bar{I}$ and the bar denotes time averaging. Thus

$$C(t) = \frac{e^2}{L^2} N_c\, \overline{\delta v_1(0)\, \delta v_1(t)} = \frac{e^2 n}{L}\, \overline{\delta v_1(0)\, \delta v_1(t)} \tag{2.3}$$

and n is the charge density per unit length. Since all particles are equivalent, by assumption, we use $i = 1$ in Eq. (2.3).

In the presence of an external field on the backbone a carrier is assumed to have probabilities p_1 and p_2 to go, respectively, to the right and to the left (the field points from left to right). For definiteness, we assume $p_1 + p_2 = 1/2$ (if there were no dangling loops, the walker has a staying probability of 1/2 per step except for endpoints). A walker on the dangling loop is assumed to have a probability of 1/4 to move to its nearest neighbor. The rhs of Eq. (2.3) is zero unless the carrier is out of the loop both at time zero and t. (We assume that, once in a dangling bond, the carrier does not contribute to conduction, i.e., it has zero velocity in the direction of the field.) If the carrier is not inside a loop at time zero and it moves one step to the right, then $v(0) = 1$. If it is not inside a loop at time t and it moves then to the right, it will contribute $v(t) = 1$. In this case a contribution of 1 to $\overline{v(0)\, v(t)}$ multiplied by the appropriate probability will be made. Similar statements are true for motion to the left at both times or for one move to the left and one move to the right.

Thus, for $t \neq 0$ a typical contribution to $\overline{v(0)\, v(t)}$ is equal to:

The probability that the carrier is not inside a bond at $t = 0$
(i.e., it is on the backbone) multiplied by
the probability that it moves on its first
step to the right (i.e., p_1) multiplied by

the conditional probability that the carrier
is on the backbone at time $t-1$ (given that
it was on the backbone at $t=0$) multiplied by
the probability that its last step is to the right (p_1) (2.4)

We have to sum over all possibilities for the first and last steps (right or left) with their appropriate signs (p_1^2, p_2^2, $-p_1 p_2$, $-p_2 p_1$). We now consider each of the probabilities in (2.4).

First, the probability for an arbitrary carrier to be on the backbone is equal to $1/m$. This can be easily proven by writing a master equation and by taking into account the stationarity of the process. Thus: The probability P_b that the carrier is on the backbone at $t=0$ is

$$P_b(t=0)=1/m \tag{2.5}$$

where m is the number of sites in a dangling loop.

Define $F(t)$ as the conditional probability that the carrier is on the backbone at time $n+t$ if it was there at time n. Thus (for $t \neq 0$)

$$\overline{v_1(0)\,v_1(t)}=\frac{(p_1-p_2)^2}{m}\,F(t-1), \qquad t \geqslant 1 \tag{2.6}$$

Obviously: $F(0)=1$. It is convenient to define $F(t)$ for negative values of t. We define

$$F(t)=\begin{cases} F(-t-2), & t \leqslant -2 \\ 1 & t=-1 \end{cases}$$

For $t=0$

$$\overline{v_1(0)^2}=\frac{p_1+p_2}{m}=\frac{1}{2m} \tag{2.7}$$

and

$$\overline{v_1(0)}=\frac{p_1-p_2}{m} \tag{2.8}$$

which define the average velocity and mobility in the sample. Combining Eqs. (2.6) and (2.7), we obtain

$$\overline{v_1(0)\,v_1(t)}=\frac{1}{m}\left[\delta_{t,0}\left(\frac{1}{2}-(p_1-p_2)^2\right)+(p_1-p_2)^2\,F(t-1)\right] \tag{2.9}$$

Note that by stationarity

$$\overline{v_1(0)\,v_1(t)}=\overline{v_1(-t)\,v_1(0)}$$

The above definition of $F(t)$ incorporates this time-reversal symmetry into Eq. (2.9). The generating function (ϕ probability in I) $\hat{F}(z)$ is given by (we assume here periodic boundary conditions or an infinite sample)

$$\hat{F} = \hat{Y}_m + \hat{Y}_m(z/2)\hat{F} \tag{2.10}$$

$\hat{Y}_m$ represents the probability of going from a backbone point back to itself (including just staying there) without visiting any other backbone point. The second term in Eq. (2.10) represents the set of paths leading from a backbone point to a different backbone point. It involves a walk inside the loop of the original point (or staying there, i.e., $\hat{Y}_m$), followed by motion of a step (to the right or to the left) on the backbone, with probability $(p_1 + p_2)z = z/2$, which is necessary to get to another point, and finally a subsequent motion to any other backbone point (given by $\hat{F}$). Solving Eq. (2.10) for $\hat{F}$, we obtain

$$\hat{F} = \frac{\hat{Y}_m}{1 - \frac{1}{2}z\hat{Y}_m} \tag{2.11}$$

We return now to the current autocorrelation function $C(t)$ [Eqs. (2.2) and (2.3)]. By stationarity $C(t) = C(-t)$. Applying the Wiener–Khinchine theorem and using Eqs. (2.3) and (2.9), we obtain the spectrum $S(\omega)$ corresponding to $C(n)$:

$$S(\omega) = \sum_{n=-\infty}^{\infty} C(n)\, e^{in\omega} \tag{2.12}$$

The contribution of F to $S(\omega)$ is

$$\sum_{t=-\infty}^{\infty} e^{i\omega t} F(t-1) = 1 + 2\,\mathrm{Re}[z\hat{F}(z)]$$

Recall that

$$\hat{F}(e^{i\phi}) = \sum_{n=0}^{\infty} e^{i\phi n} F(n)$$

(where $z = e^{i\phi}$), i.e., it is a Poisson (one-sided) transform. Hence

$$S(\omega) = \frac{e^2 n}{L}\left\{\frac{1}{2m} + \frac{(p_1 - p_2)^2}{m}\left[-1 + \sum_{n=-\infty}^{n=+\infty} F(n-1)e^{in\omega}\right] - \left(\frac{p_1 - p_2}{m}\right)^2 \cdot 2\pi\delta(\omega)\right\} \tag{2.13}$$

In (2.13) the last term is the Fourier transform of $(\bar{I})^2$ (which is subtracted from $\overline{I(t)\,I(0)}$ [see Eq. (2.2)]. Recall that $F(n) \to 1/m$ when $n \to \infty$ by its definition. Thus, the Fourier transform of $F(n)$ in Eq. (2.13) contains a part that is $(1/m)\,\delta(\omega)$. This part cancels the term containing $\delta(\omega)$ in Eq. (2.13). This fact can also be seen from a direct analysis of Eq. (2.11) [see Eq. (A.3) in the Appendix for the form of $Y_m(z)$].

The spectrum $S(\omega)$ in Eq. (2.13) can be rewritten

$$S(\omega) = \frac{e^2 n}{Lm}\left\{\frac{1}{2} + 2(p_1 - p_2)^2 \operatorname{Re}[z\hat{F}(z)] - \frac{(p_1 - p_2)^2}{m}\, 2\pi\delta(\omega)\right\} \tag{2.14}$$

Note that the nonwhite contribution in Eq. (2.13), which we define as $S_I(\omega)$, is derived directly from the generating function $F(z)$. This feature appears to be general.

In Fig. 3 a plot of $\operatorname{Re}[zF(z)] \propto S_I(\omega)$ is shown for $m = 1000$. For small ω, we observe that $S_I(\omega) \approx \omega^{-1/2}$ with a plateau ranging from $\omega \geqslant 0$ to $\omega \approx 1/m^2$. In the Appendix, we present an analytic derivation of this result using the function Y_m obtained in I. It should be noted that this behavior depends on the assumed type of side branches (or dangling bonds). For example, for the structure (blob) depicted in Fig. 2, we have $S_I(\omega) \approx \omega^{-1/4}$ (see the Appendix). Thus, we obtain a $1/f^\alpha$ behavior with $\alpha \approx 1/2$ (for the simple loops) or $\alpha = 1/4$ (for the blob). As in I, the cutoff for the power law dependence is provided by the length of the dangling loops. Here the condition is $\omega > 1/m^2$. The spectrum for the total current per unit length is

$$S_I(\omega) = 2\,\frac{e^2 nm}{L}\,\bar{v}_1^2 \operatorname{Re} z\,\hat{F}(z)|_{z = e^{i\omega}} \tag{2.15}$$

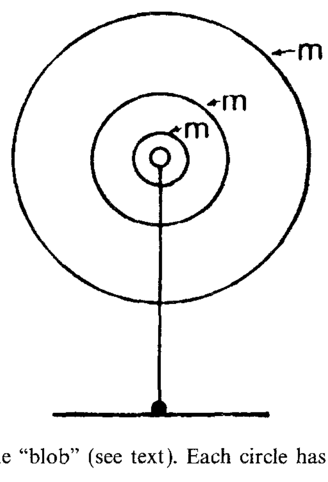

Fig. 2. The "blob" (see text). Each circle has a length m.

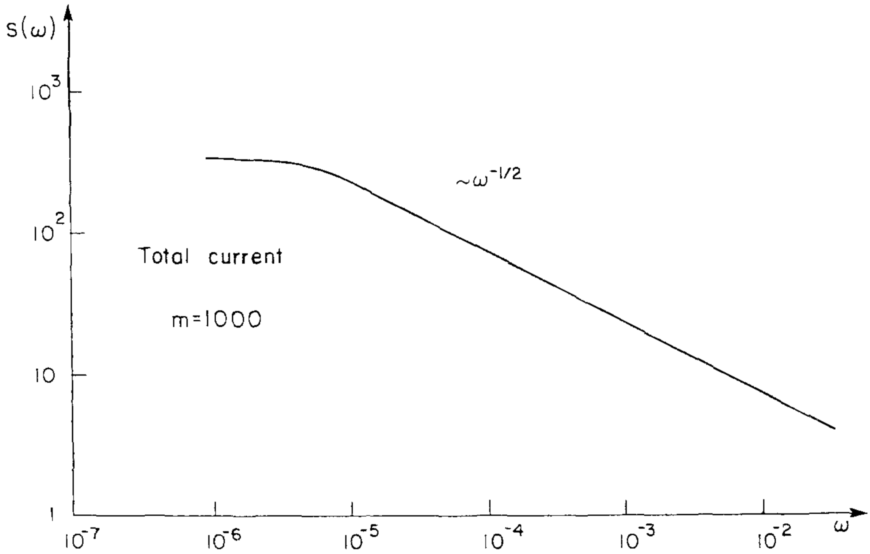

Fig. 3. Plot of $S_I(\omega)$ against ω for $m = 1000$ for the total current [see Eq. (2.14)].

where $\bar{v}_1$ is defined in Eq. (2.8). The mean waiting time in a loop α was calculated in I and was found to equal $2m - 1$. Using the definition of $\bar{I}$ in Eq. (2.1), one can write the spectrum as

$$S_I(\omega) = \frac{\bar{I}^2 \alpha}{N_c} \operatorname{Re} z \hat{F}((z)|_{z = e^{i\omega}} \tag{2.16}$$

This result resembles that of Tunaley,[6] but is of course deduced differently. Note that the prefactor $\bar{I}^2/N_c$ corresponds precisely to the experimental findings.

2.2. Current Correlations: Alternate Definitions

Although the current correlations as defined above have properties compatible with experiment, they are not precisely the quantity measured by an amperemeter attached to the ends of a system.[24] The current, in the strict sense, is measured outside the resistor and is defined as the net number of charges passing through a cross section per unit time. Our model makes possible the use of this (standard) definition for the calculation of the current correlations and the spectra. (In other hopping models we are aware of, one deals with infinite or periodic systems, and end effects on current—measured in a realistic way—are not considered). In this section we shall consider two models differing by their boundary conditions.

Consider a segment of length N (with dangling loops, as before) at the ends of which („O" and "N") there are connected two electrodes. We consider two possibilities for the boundary conditions at the electrodes.

Model 1. A Current Source at One Electrode. Assume that "0" and "N" are coupled to reservoirs, keeping the density ρ_N at point "N" constant. For simplicitly, we choose $\rho_N = 0$ ("N" os a sink). We assume that particles are injected at a constant rate ρ into the system. The average current is obviously ρ. The current at "N," however, is a fluctuating quantity since (in our model) the charges perform a random walk from "0" to "N." Define a set $\{X\}$ of random variables:

$$X_{i,\tau}(t), \qquad i = 1, 2, ..., \rho, \quad \tau = -\infty, ..., t \tag{2.17}$$

associated with the ith particle which was injected at time τ and whose presence at "N" is measured at time t (which is also its time of first arrival, since "N" is a sink by assumption). If present, this particle contributes one unit of current measured at "N" and $X_{i,\tau}(t) = 1$; otherwise, $X_{i,\tau}(t) = 0$. The instantaneous current at "N" is

$$I_N(t) = \sum_{\tau < t} \sum_{i=1}^{\rho} X_{i,\tau}(t) \tag{2.18}$$

and

$$\begin{aligned} X_{i,\tau}(t) &= 1 \quad \text{with probability } G_N(t-\tau) \\ X_{i,\tau}(t) &= 0 \quad \text{with probability } 1 - G_N(t-\tau) \end{aligned}$$

where $G_N(t)$ is the first passage probability distribution, as defined in I. From Eq. (2.2) the current autocorrelation function is

$$C(t) = \overline{I(0)\, I(t)} - \rho^2 \tag{2.19}$$

By Eq. (2.18) we have

$$C(t) = \sum_{i,\tau} \sum_{i',\tau'} \overline{X_{i,\tau}(0)\, X_{i',\tau'}(t)} - \rho^2 \tag{2.20}$$

The summand in Eq. (2.20) decouples:

$$\overline{X_{i,\tau}(0)\, X_{i',\tau'}(t)} = \overline{X_{i,\tau}(0)}\; \overline{X_{i',\tau'}(t)} \tag{2.21}$$

except for $i = i'$ and $\tau = \tau'$ (since probabilities for different particles are, by assumption, independent). In the latter case

$$\overline{X_{i,\tau}(0)\, X_{i,\tau}(t)} = \delta_{t,0} \cdot G_N(t-\tau) \tag{2.22}$$

Hence

$$C(t) = \sum_{i \neq i' \text{ or } \tau \neq \tau'} \overline{X_{i,\tau}(0)}\ \overline{X_{i',\tau'}(t)} + \sum_{i,\tau} \overline{X_{i,\tau}(0)\, X_{i,\tau}(t)} - \rho^2 \qquad (2.23a)$$

Thus

$$C(t) = \sum_{i,i',\tau,\tau'} \overline{X_{i,\tau}(0)}\ \overline{X_{i',\tau'}(t)} - \sum_{i,\tau} \overline{X_{i,\tau}(0)}\ \overline{X_{i,\tau}(t)} + \sum_{i,\tau} \overline{X_{i,\tau}(0)\, X_{i,\tau}(t)} - \rho^2 \qquad (2.23b)$$

Since

$$\overline{X_{i,\tau}(t)} = G_N(t-\tau)$$

and

$$\overline{X_{i,\tau}(t)} = 0 \qquad \text{for} \quad t < \tau$$

by its definition, and since $\sum_{n=0}^{\infty} G_N(n) = 1$ (see Refs. 20 and 21), we obtain, using Eq. (2.22),

$$C(t) = \rho\delta_{t,0} - \rho \sum_{n=0}^{\infty} G_N(n)\, G_N(n+t) \qquad (2.23c)$$

The absolute value in Eq. (2.23c) comes from stationarity. Note that the ρ^2 term in Eq. (2.19) is cancelled by the first term on the rhs of Eq. (2.23b), which is fully decoupled. It is interesting to note that for nonzero times the correlation $C(t)$ is negative. The reason for this feature is the fact that a positive fluctuation of the current at "N" leads to a depletion of "charge" near "N," thus making the subsequent current less than its average. A similar argument holds for a negative fluctuation. This process can be regarded as a result of the "tendency" of the system to restore the average current following fluctuations. The spectrum $S(\omega)$ corresponding to Eq. (2.23c) is

$$S(\omega) = \rho - \rho \sum_{t=-\infty}^{\infty} e^{i\omega t} \sum_{n=0}^{\infty} G_N(n)\, G_N(n+t) \qquad (2.24)$$

The last term in Eq. (2.24) can be evaluated as follows. It is shown in Appendix B of I that for the system with which we deal here, $G_N(n)$ is of the form $Cn^{-3/2}$ in the range $m^2 \gg n \gg N$ (C being a constant). For $n \gg m^2$, the decay of $G_N(n)$ is exponential and for n between N and εN^2 (ε is a real number satisfying $1/N \ll \varepsilon \ll 1$), $G_N(n)$ rises steeply from zero

$[G_N(N-1)=0]$. Thus, in order to compute the sum in Eq. (2.23), we shall model $G_N(n)$ in the following way:

$$G_N(n)=\begin{cases} 0, & n<\varepsilon N^2 \\ Cn^{-3/2}, & \varepsilon N^2<n<m^2 \\ 0, & m^2<n \end{cases}$$

Substituting this equation into the sum and approximating the sum in Eq. (2.24) by an integral, we obtain

$$C(t)\approx -C\rho^2\int_{\varepsilon N^2}^{m^2}\frac{dn}{[n(n+t)]^{3/2}}+\rho$$

Defining $n=\beta t$, we have

$$\begin{aligned} C(t)&\approx\frac{-C\rho^2}{t^2}\int_{\varepsilon N^2/t}^{m^2/t}\frac{d\beta}{[\beta(\beta+1)]^{3/2}}+\rho \\ &\approx\frac{-CC\rho^2}{t^2}\int_{\varepsilon N^2/t}^{\delta}\frac{d\beta}{\beta^{3/2}}-\frac{C\rho^2}{t^2}\int_{\delta}^{m^2/t}\frac{d\beta}{[\beta(\beta+1)]^{3/2}}+\rho \end{aligned}$$

where $\varepsilon N^2/t\ll\delta\ll 1$. When $m^2/t\gg 1$ we obtain

$$C(\tau)\approx\frac{2C\rho^2}{t^2}\left[\delta^{-1/2}-\frac{1}{N}\left(\frac{t}{\varepsilon}\right)^{1/2}\right]-\frac{C\rho^2}{t^2}k(\delta)+\rho$$

where $k(\delta)$ is a constant. Thus, the leading term for small ω (but $\omega\gg 1/m^2$) in the Fourier transform of $C(\tau)$ is of the type $-A\sqrt{\omega}$ with A a constant. Hence

$$S(\omega)\approx\rho-\frac{2C\rho^2}{N\sqrt{\varepsilon}}\sqrt{\omega}$$

The spectrum we have obtained contains a nonanalytic negative and non-white part. This result should be contrasted with those of Nieuwenhuizen and Ernst[25] and Lehr *et al.*,[26] in which $S(\omega)\approx\text{const}+\omega^{-1/2}$, when the total current is investigated. It would be interesting to find a system that exhibits this kind of behavior.

We now present a more realistic system, which has feedback.

Model B. A System with Positive Feedback. The system is shown in Fig. 3. A particle arriving at "N" enters the "electrode" and is immediately transferred (on the next step) through a resistanceless battery to "electrode" "0." It seems to us that this system is more realistic than

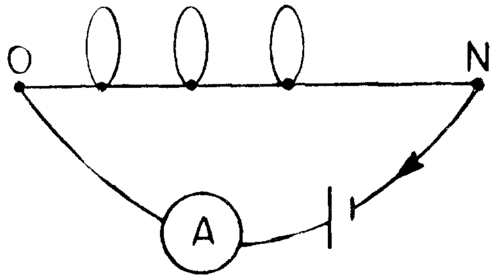

Fig. 4. The segment with positive feedback. The current is "measured" in the "amperemeter" A outside the system.

model A. The outgoing current is directly related to the incoming current, thereby producing feedback. The role of the battery is to create the bias inside the resistor by keeping the voltage constant. The number of carriers N_c is obviously kept constant. The system is periodic, with the exception of two sites. At "0" the probability of returning to "N" is zero, while at "N" the probability to get to "0" is unity. Define, as before, a set of random variables $\{Y\}$ as follows:

$$Y_{i,j}(t)=\begin{cases}1 & \text{if the } i\text{th particle } (1\leqslant i\leqslant N_c) \text{ iq at "}j\text{" at time } t\\ 0 & \text{otherwise}\end{cases} \tag{2.25}$$

The current measured by the amperemeter at time t is thus

$$I(t)=\sum_{i=1}^{N_c} Y_{,N}(t) \tag{2.26}$$

The correlation function $C(t)$ is

$$C(t)=\sum_{i=1}^{N_c}\sum_{i''=1}^{N_c}\overline{Y_{i,N}(0)\,Y_{i',N}(t)}-\bar{I}^2 \tag{2.27}$$

Since we are dealing with noninteracting particles, all the terms in Eq. (2.27) decouple except the one with $i=i'$, for which we may write

$$\sum_{i=1}^{N_c}\overline{Y_{i,N}(0)\,Y_{i,N}(t)}$$
$$=\bar{I}\delta_{t,0}+N_c$$
$$\times\text{(Probability that carrier } i \text{ is at "}N\text{" at } t=0)$$
$$\times\text{[Probability of a transition from "}N\text{" to "0" in one step (which equals 1)]}$$
$$\times\text{[Probability of a transition from "0" to "}N\text{" in } t-1 \text{ steps (not necessarily for the first time)]}$$

Hence

$$\sum_{i=1}^{N_c} \overline{Y_{i,N}(0)\, Y_{i,N}(t)} = \bar{I}\delta_{t,0} + N_c P(N,0)\, P(N, t-1; N, 0) \tag{2.28}$$

where $P(N, 0)$ is the probability to be at "N" at time 0 and $P(N, t; N, 0)$ is the (conditional) probability of a transition from "0" to "N" in t steps. [The reason for the appearance of $t-1$ in Eq. (2.28) is the fact that one step is "wasted" for moving from "N" to "0."] Substituting (2.28) in (2.27), we obtain

$$C(t) = \sum_{i \neq i'} \overline{Y_{i,N}(0)}\ \overline{Y_{i',N}(t)} + \sum_i \overline{Y_{i,N}(0)\, Y_{i,N}(t)} - \bar{I}^2 \tag{2.29a}$$

Note that for $i \neq i'$, the average in Eq. (2.29a) is decoupled (independent particles). Hence

$$C(t) = \sum_{i,i'} \overline{Y_{i,N}(0)}\ \overline{Y_{i',N}(t)} - \sum_i \overline{Y_{i,N}(0)}\ \overline{Y_{i,N}(t)}$$
$$+ \sum_i \overline{Y_{i,N}(0)\, Y_{i,N}(t)} - \bar{I}^2 \tag{2.29b}$$

Now, from Eq. (2.26)

$$\sum_{i,i'} \overline{Y_{i,N}(0)}\ \overline{Y_{i',N}(t)} = \bar{I}^2$$

Also, by the independence of the walkers,

$$\sum_i \overline{Y_{i,N}(0)}\ \overline{Y_{i,N}(t)} = N_c\ \overline{Y_{1,N}(0)}\ \overline{Y_{1,N}(t)}$$

or

$$\sum_i \overline{Y_{i,N}(0)}\ \overline{Y_{i,N}(t)} = \bar{I}^2/N_c$$

Substituting the above results in (2.29a), we obtain

$$C(t) = \sum_i \overline{Y_{i,N}(0)\, Y_{i,N}(t)} - \bar{I}^2/N_c \tag{2.29c}$$

Substituting Eq. (2.28) into Eq. (2.29c), we finally obtain

$$C(t) = \bar{I}\delta_{t,0} + N_c P(N,0)\, P(N, t-1; N, 0) - \bar{I}^2/N_c \tag{2.29d}$$

Obviously, $N_c P(N, 0) = \bar{I}$. It remains to calculate $P(N, t-1; N; 0)$. Its generating function $\hat{H}_N(z)$ is found by the following argument.

A particle located at "N" goes to "0" at its first step with probability 1, i.e., the corresponding ϕ probability equals z. It then goes from "0" to "N," for the first time, with ϕ probability $\hat{G}_N(z)$. The process can be repeated indefinitely so that

$$\hat{H}_N(z) = z\hat{G}_N(z) + [z\hat{G}_N(z)]^2 + \cdots \tag{2.30}$$

and

$$\hat{H}_N(z) = \frac{z\hat{G}_N(z)}{1 - z\hat{G}_N(z)} \tag{2.31}$$

The spectrum $S(\omega)$ is then [see Eq. (2.12)]

$$S(\omega) = \bar{I} - \frac{\bar{I}^2}{N_c}\,\delta(\omega) + 2\bar{I}\,\mathrm{Re}\,\hat{H}(e^{i\omega}) \tag{2.32}$$

The term $\delta(\omega)$ is calceled by a similar term arising from $\mathrm{Re}\,H(e^{i\omega})$ (see below for the structure of H), as happened in Eq. (2.13) above.

In Appendix X of I it is shown that the generating function $G_N(z)$ in the case of infinite bias is

$$\hat{G}_N(z) = [(\tfrac{1}{2}\hat{Y}_m(z)]^N \tag{2.33}$$

Hence in this case

$$\hat{H}_N(z) = \frac{z[\frac{1}{2}\hat{Y}_m(z)]^N}{1 - z[\frac{1}{2}\hat{Y}_m(z)]^N} \tag{2.34}$$

In the Appendix we show that in this case $\mathrm{Re}\,H(e^{i\omega}) \approx \omega^{-1/2}$ for small ω that are still larger than $1/m^2$. A similar power law is presented in Refs. 25 and 26. Figure 5 is a plot of $\mathrm{Re}\,\hat{H}(e^{i\omega})$ for $N = 10$ and $m = 400$. In Fig. 5, $\mathrm{Re}\,\hat{H}(e^{i\omega})$ is plotted in the case of zero, intermediate, and extremely large bias ($p_1 = 0.499$), $p_2 = 0.001$). We use the exact expression (C.14) in I for $\hat{G}_N$ valid for arbitrary bias. Only in the case of strong bias does one observe a clear power law behavior. (In I, bias is defined as $p_1 \propto e^b$ and $p_2 \propto e^{-b}$, $p_1 + p_2 = 1/2$.)

In experiments,[15] the observed "$1/f$" spectra are proportional to $\bar{I}^2/N_c$. To understand this fact, note that $N_c P(N, 0) = \bar{I}$, $P(N, 0) = 1/N$ (homogeneous distribution), and thus $\bar{I}/N = \bar{I}^2/N_c$. Moreover, $\hat{H} \propto 1/N$, as shown in the Appendix (for infinite bias). Hence, from Eq. (2.32)

$$\bar{I}\,\mathrm{Re}\,\hat{H}(e^{i\omega}) \propto \bar{I}^2/N_c$$

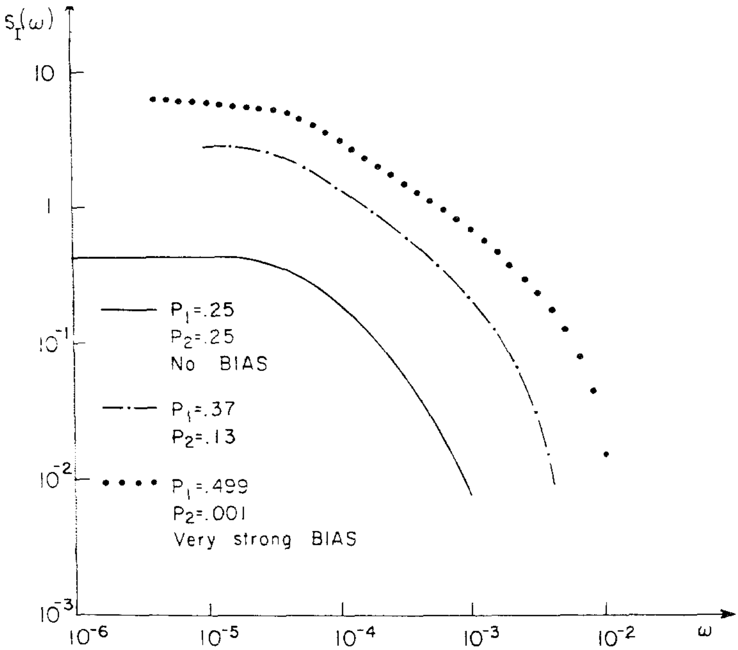

Fig. 5. Plots of Re $H(e^{i\omega})$ [$\propto S_I(\omega)$] for $N=10$, $m=400$ for three values of blas: $p_1=0.25$ (no bias), $p_1=0.37$, $p_1=0.499$ [see Eq. (2.32)].

as observed experimentally. This result is correct in the limit of large bias. Thus, the strong bias in experiments is not only necessary to "overcome" other noises in the system. In our model (and perhaps in reality) the strong bias is necessary in order to have the effect in the first place.

3. EFFECT OF THE DANGLING BOND DISTRIBUTION

To make our model more realistic it is certainly appropriate to introduce a distribution $P(m)$ of lengths m of the dangling bonds.

The generating function for first passage $\hat{G}_N(z)$ averaged over all possible configurations $\{m\}$ $(m_0, m_1, \ldots, m_{N-1})$ of the lengths of the dangling bonds reads

$$\hat{G}_N(z) = \sum_{\{m\}} P(\{m\})\, \hat{G}_N(z, \{m\}) \tag{3.1}$$

where $G_N(z, \{m\})$ is the generating function for realization $\{m\}$ and (assuming the different m_i to be statistically independent)

$$P(\{m\}) = \prod_{i=0}^{N-1} P(m_i) \tag{3.2}$$

Define

$$\hat{Y}(z) = \sum_{\{m\}} P(m)\, \hat{Y}_m(z) \tag{3.3}$$

$\hat{Y}(z)$ can be calculated once $P(m)$ is known. In the case of infinite bias

$$\hat{G}_N(z) = \left(\frac{z}{2}\right)^N \sum_{\{m\}} \prod_{i=0}^{N-1} P(m_i)\, \hat{Y}_{mi} = \left(\frac{z}{2}\,\hat{Y}\right)^N \tag{3.4}$$

Define $\hat{F}_i(z, \{m\})$ to be the generating function for motion from site "i" to any other site on the backbone. $\hat{F}_i$ is the generalization of $\hat{F}(z)$ defined in Eq. (2.10) for the case of loops of uniform lengths. $\hat{F}_i$ is given (for any bias) by

$$\hat{F}_i = \hat{Y}_{m_i} + \hat{Y}_{m_i} p_1 z \hat{F}_{i+1} + \hat{Y}_{m_i} p_2 z \hat{F}_{i-1} \tag{3.5}$$

In Eq. (3.5) the first term stands for a walk in which the particle does not leave site "i" to another site in the backbone. The two other terms represent a walk in which the carrier enters the loop as many times as it wishes, then does a step to the left (right) and at this site the generating function to move to another site on the backbone is $\hat{F}_{i-1}$ (or $\hat{F}_{i+1}$). Defining the configuration average of $\hat{F}_i$ by

$$\hat{F} = \sum_{\{m\}} P_{m_i} \hat{F}_i(z, \{m\})$$

and approximating

$$\overline{Y_{m_i}(z)\, F_i(z, \{m\})} = \overline{Y_{i,N}(z)}\; \overline{F_i(z, \{m\})}$$

we obtain

$$\hat{F} = \hat{Y} + \hat{Y}(z/2)\hat{F} \tag{3.6}$$

for an infinite system (this is a mean-field-like approximation). In order to proceed, we need to know the distribution $P(\{m\})$.

The commonly accepted picture that emerges from percolation theory is that conduction is limited to the backbone, a substructure of the percolation cluster. Most of the mass of the cluster is concentrated in the dangling bnds, which by definition do not participate in the average conduction. It is reasonable to assume that the dangling bond lengths have a power-law distribution $P(m)$ up to a cutoff m_c. Thus,

$$\begin{aligned} P(m) &\propto m^{-x}, \qquad & m &< m_c \\ P(m) &\approx 0, \qquad & m &\geqslant m_c \end{aligned} \tag{3.7}$$

We have used Eq. (3.3) to compute $S(\omega)$ as given by Eq. (2.32). As before, we have substituted in Eq. (2.31) the generating function, $\hat{G}_N$, as given in Appendix C of I. We have replaced $\hat{Y}_m$ by its configuration average $\hat{Y}$. Figures 6 and 7 present Re $\hat{H}(e^{i\omega})\alpha\ S_I(\omega)$ versus ω for several values of x ($x=0.1$, 0.9, 1.7, assuming strong bias, i.e., $p_1=0.444$ in Fig. 6; and $x=2.1$ and 2.3, assuming very strong bias, $p_1=0.499$, in Fig. 7). In both figures $N=10$ and the cutoff is $m_c=1000$. (Both figures correspond to the system with feedback depicted in Fig. 4.) It can be seen that $S_I(\omega)$ exhibits a region in which a power law behavior is observed. For $x \geqslant 2.3$ no power law regime is obtained. In the Appendix we study analytically the following two types of spectra: the first is for the case of infinite bias in the system with feedback (see Fig. 4) and the second is the total current spectrum [on the basis of Eq. 2.16)]. In both cases we show that $S_I(\omega) \propto \omega^{-x/2}$ with a *breakdown* of the power law behavior for $x>2.26$. For $x=2$ the power spectrum is exactly "$1/f$" like. Thus, in our model there can be *no* $1/f^{\alpha}$

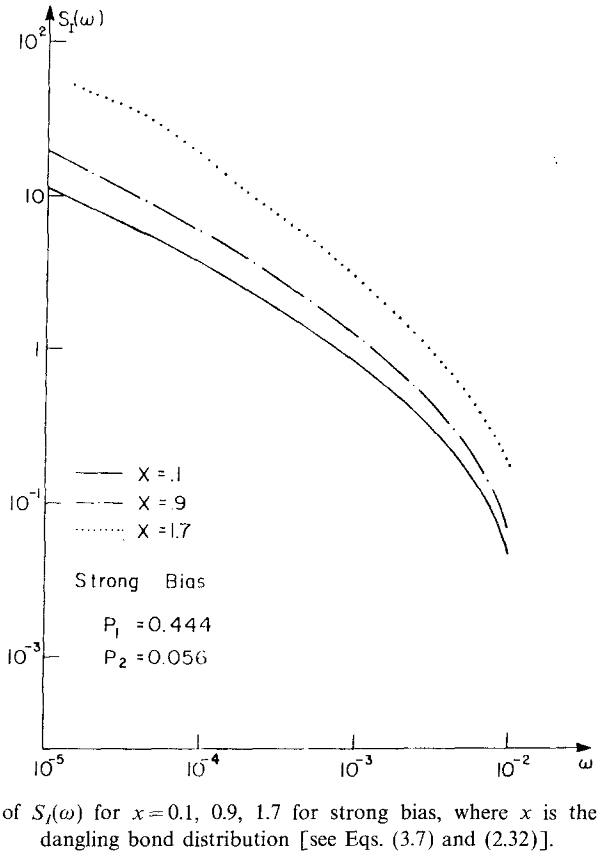

Fig. 6. Plots of $S_I(\omega)$ for $x=0.1$, 0.9, 1.7 for strong bias, where x is the parameter of dangling bond distribution [see Eqs. (3.7) and (2.32)].

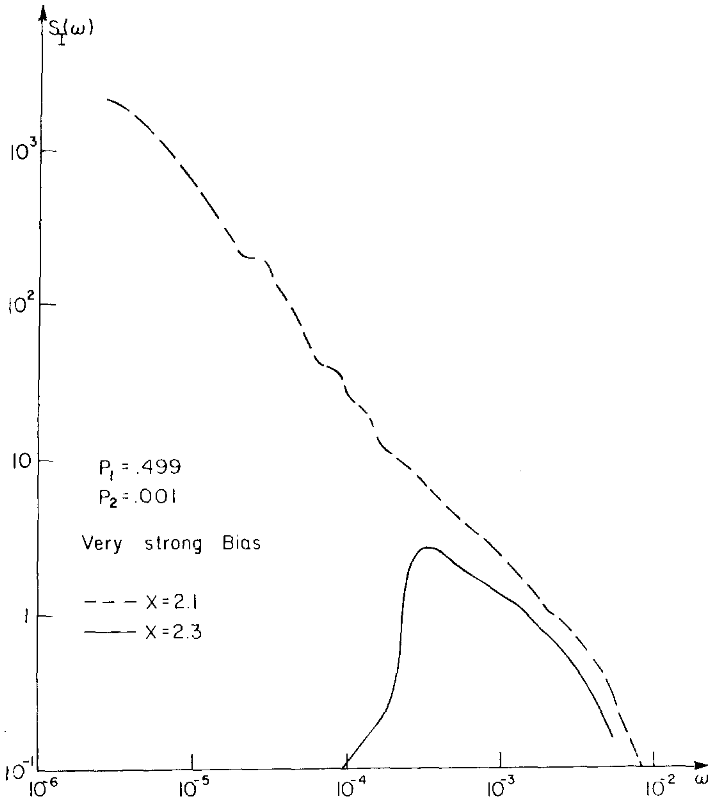

Fig. 7. Plots of $S_I(\omega)$ for $x = 2.1$ and 2.3 in the case of very strong bias. The break in the curve also occurs for low values of vias.

spectrum with $\alpha > 1.13$ (see Appendix). The generality of this result remains to be investigated. According to experiments,[15] $\alpha < 1.4$. The relevant value of x for a percolation cluster or other systems is not yet known. If one assumes that $S_I \propto \omega^{-x/2}$ is of general validity, it follows that $x \approx 2$ for most systems, i.e., $P(m) \propto m^{-2}$. Since, as we have shown before, $2m - 1$ is the mean time spent in a cluster [see Eq. (4.10) in I], it follows that the probability distribution for the mean time τ spent in a side branch (or a trap) $\bar{P}$ scales like $\bar{P} \propto \tau^{-2}$ (see also Ref. 25, but compare to $\bar{P} \approx \tau^{-1}$ in Refs. 15 and 27).

The validity of this result is under investigation.

4. SUMMARY AND CONCLUSIONS

In this investigation we have exhibited a mthod for computing current fluctuations and the corresponding spctra. We have shown how power laws emerge in the spectra as a result of the existence of dangling bonds. The role of bias in the production of these power laws has been elucidated. Among our results, we wish to mention the appearance of "$1/f$" noise, including a prefactor that is in agreement with experimental findings. We have seen the role of the length of the dangling bond as a provider of a cutoff for the power law behavior. In some cases we have observed the crossover between regions in the spectrum characterized by different power laws. It is amusing to note that, when one considers the total current in the system, a "$1/f$"-like spectrum is possible even in the presence of weak bias. The current, as measured by a "device" connected to the extremes of the network, has been shown to have a power law region in the spectrum only in the presence of strong enough bias. When a distribution of dangling bond lengths is considered, the nature of the resulting power laws in the spectrum depends directly on the nature of the assumed distribution. Having in mind percolation networks, we considered a power law distribution for the length of the dangling bonds. The resulting power spectrum for the current was directly related to the exponent appearing in the dangling bond distribution. This result suggests, at least to some extent, that "$1/f$" noise is not really a universal quantity (unless we define the power law characterizing a distribution of dangling bonds as a parameter determining a universality class). An interesting feature is the observation that when the exponent characterizing the distribution of bond lengths exceeds 2.26, "$1/f$" behavior is no longer possible in our model.

The extent to which this result is general remains to be seen.

Finally, we wish to comment on the fact that our model is basically one-dimensional. It seems that in percolating clusters, as well as in other random systems, the backbone is basically composed of one-dimensional or quasi-one-dimensional "channels," which are located quite apart from each other. Thus, in spite of the one-dimensionality of our model, it may be more realistic than may appear at first glance. Moreover, our results seem to be in agreement with those of numerical simulations done on percolating clusters and with experiments exhibiting "$1/f$" noise (e.g., the prefactor of the power law) and with diffusion experiments in porous media.

The investigation of current fluctuations in higher dimensional models is of course a worthwhile project, which we are attempting at present.

APPENDIX

A1. Asymptotic Behavior of the Spectrum for Small ω

In this Appendix the term spectrum refers to the nonwhite (i.e., nonconstant) part of the spectrum. The total spectrum is clearly positive. The nonwhite part, which is computed below, can be negative.

We first note from Eq. (2.14) that the spectrum of the total current correlation is proportional to Re$[zF(z)]$. Using Eq. (2.11), we have

$$S(\omega) \propto \operatorname{Re} \left. \frac{2/z}{(\frac{1}{2}z\hat{Y}_m)^{-1} - 1} \right|_{z=e^{i\omega}} \tag{A.1}$$

The spectrum of the current through the endpoints (the system with feedback) is given in Eq. (2.32). The spectrum is proportional to Re $\hat{H}(e^{i\omega})$. In the case of infinite bias (in practice very large) we use Eq. (2.34) and we obtain

$$S(\omega) \propto \operatorname{Re} \left. \frac{1}{z^{-1}(\frac{1}{2}z\hat{Y}_m)^{-N} - 1} \right|_{z=e^{i\omega}} \tag{A.2}$$

Note that if we set $N=1$ in Eq. (A.2) we recover the spectrum in Eq. (A.1) [apart from the factor z^{-1} $(z \approx 1 + i\omega)$, which equals unity for small frequencies, and thus does not influence the asymptotic results]. Thus, we shall concentrate on the spectrum given by Eq. (A.2), since the spectrum of the total current autocorrelation is the special case of it.

From paper I [see Eq. (A.11) in I] we have

$$\hat{Y}_m(e^{i\omega}) = \frac{2e^{-i\omega}}{1 - \sin\theta \operatorname{tg}(m\theta/2)} \tag{A.3}$$

where

$$e^{i\omega} = \frac{1}{\cos^2(\theta/2)} \tag{A.4}$$

For $\omega \approx 0$, $\theta = 2(i(\omega))^{1/2}$. In the same limit

$$\operatorname{tg}\frac{m\theta}{2} \approx \frac{\sin[m(2\omega)^{1/2}] + i\sinh[m(2\omega)^{1/2}]}{\cos[m(2\omega)^{1/2}] + \cosh[m(2\omega)^{1/2}]} \tag{A.5}$$

When $m\sqrt{\omega} \gg 1$ and $\omega \approx 0$, then $\operatorname{tg}(m\theta/2) \approx i$ from Eq. (A.5). Hence

$$\hat{Y}_m \approx \frac{2e^{-i\omega}}{1 - 2i(i\omega)^{1/2}} \tag{A.6}$$

On the other hand, for $m\sqrt{\omega} \ll 1$,

$$\hat{Y}_m \approx \frac{2e^{-i\omega}}{1 - 2im\omega} \tag{A.7}$$

Using Eqs. (A.2) and (A.6), we obtain for the spectrum in the case of infinite bias

$$S(\omega) \propto \frac{1}{2\sqrt{2}N\omega^{1/2}} \qquad \text{with} \quad \frac{1}{m^2} < \omega < \frac{1}{N^2} \tag{A.8}$$

The upper limit for the validity of Eq. (A.8) follows from the fact that in Eq. (A.2) we Taylor-expand an expression raised to the Nth power. Therefore, in the same limit, it follows from Eqs. (A.1) and (A.6) that the spectrum corresponding to the total current, for $m\sqrt{\omega} \gg 1$, is

$$S(\omega) \propto \frac{1}{2\sqrt{2}\omega^{1/2}}$$

Here, the range of validity is as in Eq. (A.8), where one substitutes $N = 1$. When $\omega \ll m^2$ the real part of both spectra vanishes, as can be seen by substituting $\hat{Y}_m$ from Eq. (A.7).

A2. The Dangling Blob

In I we introduced the blob, i.e., a loop-within-loop structure (see Fig. 2). $\hat{Y}_b(m)$, the analog of $\hat{Y}_m$, is [see Eq. (A.30) in I]

$$\hat{Y}_b(m) = \frac{1}{1 - (z/2)^2 \hat{Y}_m \{1 \pm [1 - (z\hat{Y}_m/2)^2]^{1/2}\}^{-1}} \tag{A.9}$$

The choice of sign in Eq. (A.9) depends on Γ [see Eqs. (A.23)–(A.27) in I]:

$$\Gamma = \frac{1 - [1 - (z\hat{Y}_m/2)^2]^{1/2}}{1 + [1 - (z\hat{Y}_m/2)^2]^{1/2}} \tag{A.10}$$

For small ω but $m\sqrt{\omega} \gg 1$ we have, using Eq. (A.6),

$$\Gamma \approx \frac{1 - [-2i(2i\omega)^{1/2}]^{1/2}}{1 + [-2i(2i\omega)^{1/2}]^{1/2}} \approx 1 + 2^{7/4} e^{i\pi 7/8} \omega^{1/4} \tag{A.11}$$

In this limit, $\Gamma < 1$ and we must choose the positive sign in Eq. (A.9). Thus, using (A.6), we obtain from (A.9), in the same limit,

$$\hat{Y}_b(m) \approx 2\{1 - [-2i(2i\omega)^{1/2}]^{1/2}\} \tag{A.12}$$

The spectrum of the current in the infinite-bias case [see Eq. (A.2)] is

$$S(\omega) \propto \mathrm{Re}\, \frac{1}{e^{-(N+1)i\omega}\{1 + N[-2i(2i\omega)^{1/2}] - 1\}}$$

$$S(\omega) \propto \frac{1}{N} \mathrm{Re}\, \frac{1}{[-2i(2i\omega)^{1/2}]^{1/2}} \propto \frac{1}{N} \cos\left(\frac{\pi}{4}\right) \omega^{-1/4}$$

Once again we obtain a power law behavior.

When $m\sqrt{\omega} \ll 1$ we obtain from Eq. (A.7)

$$\Gamma \approx \frac{1-[1-1/(1-im\omega)^2]^{1/2}}{1+[1-1/(1-im\omega)^2]^{1/2}} \approx 1-2^{3/2}\exp[(i\pi)^{3/4}(m\omega)^{1/2}]$$

Here $\Gamma > 1$ and we must choose the negative sign in Eq. (A.9). Hence, in this limit

$$\hat{Y}_b(m) \approx \frac{1}{1-\frac{1}{2}e^{i\omega}[(1-im\omega)-(-2im\omega)^{1/2}]^{-1}}$$

or

$$\hat{Y}_b(m) \approx 2[1+(-2im\omega)^{1/2}]$$

Substituting this result in Eq. (A.2), we obtain

$$S(\omega) \propto \mathrm{Re}\,\frac{1}{e^{-i\omega(N+1)}[1+(-2im\omega)^{1/2}]^{-N}-1}$$

$$\approx \mathrm{Re}\,\frac{1}{-N(-2im\omega)^{1/2}}$$

$$S(\omega) \propto \frac{1}{N\sqrt{m}}\,\omega^{-1/2}$$

Thus, when $\omega \gg 1/m^2$ we have $S(\omega) \propto \omega^{-1/4}$ and when $1/m^2 \gg \omega$ it follows that $S(\omega) \approx \omega^{-1/2}$ (in both limits we must have $N\sqrt{\omega} \ll 1$). It is interesting to note that in this case the power law does not have a lower cutoff. This is due to the fact that the blob is infinite. Note that in I it is shown that the waiting time distribution of the blob has a power law decay $n^{-5/4}$. It can be shown that for dangling bond structures with waiting time distribution $\Psi_m(n) \propto n^{-(1+\alpha)}$ then $S(\omega) \propto \omega^{-\alpha}$. This can be deduced from a Tauberian argument. We recall that $\hat{Y}_m$ is defined as the Poisson transform of $\Psi_m(n)$. If $\Psi_m(n)$ is given by the power law just described, one obtains from a Tauberian argument $\hat{Y}_m \propto 1+\kappa\omega^\alpha$. In the limit $\omega \to 0$, Eq. (A.2) yields $S(\omega) \propto \omega^{-\alpha}$ by a derivation similar to the one presented here.

A3. Distribution of Dangling Bonds

The Y function [see Eq. (3.3)] is

$$\hat{Y}(e^{i\omega}) = \frac{e^{i\omega}}{2}\kappa\int_1^{m_c}\frac{m^{-x}}{1-\sin\theta\,\mathrm{tg}(m\theta/2)}\,dm \tag{A.13}$$

where m_c is a cutoff and κ is the probability normalization.

For ω small and $m\sqrt{\omega} > 1$, Eq. (A.13) reads

$$2e^{-i\omega}\hat{Y}(e^{i\omega}) \approx 1 + \kappa\theta \int_1^{m_c} (m^{-x}) \operatorname{tg} \frac{m\theta}{2} \, dm \tag{A.14}$$

We consider two regions: $m\theta/2 \ll 1$ and $m\theta/2 \gg 1$. In the first region $\operatorname{tg}(m\theta/2) \approx m\theta/2$, whereas in the second $\operatorname{tg}(m\theta/2) \approx i$. Thus, from Eq. (A.14)

$$2e^{-i\omega}\hat{Y}(e^{i\omega}) \approx 1 + \frac{\kappa\theta^2}{2} \int_1^{2/\theta} m^{1-x} \, dm + \kappa\theta i \int_{2/\theta}^{m_c} m^{-x} \, dm \tag{A.15}$$

Hence

$$2e^{-i\omega}\hat{Y}(e^{i\omega}) \approx 1 + \frac{\kappa 2^{1-x}}{(2-x)(x-1)} [(x-1) + i(2-x)]\theta^x - \frac{\kappa}{2(2-x)}\theta^2 \tag{A.16}$$

where we have neglected the term in m_c^{1-x}, assuming $x > 1$ (see text). Note that from Eq. (A.16) one has that the waiting time distribution behaves like $t^{-1-(x/2)}$ for large t. In this limit, $S(\omega)$ for infinite bias [see Eq. (A.2)] is

$$S(\omega) \approx \frac{1}{N\kappa} \operatorname{Re} \frac{2-x}{[2^{1-x}/(1-x)][(x-1) + i(2-x)]\theta^x + \theta^2/2} \tag{A.17a}$$

We distinguish two cases, $x < 2$ and $x \geq 2$. In the first case the first term in the denominator of (A.14) dominates in the limit $\theta \to 0$ and

$$S(\omega) \approx \frac{1}{N\kappa} 2^{x-1}(2-x)(x-1) \operatorname{Re} \frac{-\theta^{-x}}{(x-1) + i(2-x)} \tag{A.17b}$$

Using $\theta \approx 2(i\omega)$ for small ω, we have

$$S(\omega) \approx \frac{1}{2N\kappa} \frac{(2-x)(x-1)}{(x-1)^2 + (2-x)^2} A(x)\omega^{-x/2} \tag{A.18}$$

where

$$A(x) = (2-x)\sin(\pi x/4) - (x-1)\cos(\pi x/4) \tag{A.19}$$

and $A(x) > 0$ for $x < 2$, as it should. When $x > 2$, the term in θ^2 in the denominator dominates. This term is imaginary and will not change the sign of the spectrum in Eq. (A.17a). At $x = 2$, $A(x)$ changes sign and becomes negative. For $x > 2$, $S(\omega)$ stays positive because of the prefactor $(2-x)$ in Eq. (A.17b). There is, however, a value x for which the spectrum becomes negative. This value is found as the solution of the equation

$$\operatorname{tg} \frac{\pi x}{4} = \frac{x-1}{2-x} \tag{A.20}$$

[i.e., $A(x) = 0$]. The solution is $x \approx 2.26$. Thus, $x = 2.26$ is the limit value above which *no* "$1/f$" noise is possible in our model.

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